

Color separate singlets in e^+e^- annihilation

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Abstract

We use the method of color effective Hamiltonian to study the properties of states in which a gluonic subsystem forms a color singlet, and we will study the possibility that such a subsystem hadronizes as a separate unit. A parton system can normally be subdivided into singlet subsystems in many different ways, and one problem arises from the fact that the corresponding states are not orthogonal. We show that if only contributions of order $1/N_c^2$ are included, the problem is greatly simplified. Only a very limited number of states are possible, and we present an orthogonalization procedure for these states. The result is simple and intuitive and could give an estimate of the possibility to produce color separated gluonic subsystems, if no dynamical effects are important. We also study with a simple MC the possibility that configurations which correspond to "short strings" are dynamically favored. The advantage of our approach over more elaborate models is its simplicity, which makes it easier to estimate color reconnection effects in reactions which are more complicated than the relatively simple e^+e^- annihilation.

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I. INTRODUCTION

Hadronic processes in high energy reactions are generally formulated in terms of two separate phases, one initial perturbative phase described by quarks and gluons followed by a nonperturbative hadronization phase. In the first phase the probability to produce a specific parton state, with specified colors, can be calculated by perturbative quantum chromodynamics(PQCD). The transition from a partonic state to a hadronic one is a nonperturbative phenomenon and can only be described by hadronization models. Hence there is an intermediate interface between the perturbative stable partonic state and the hadronic one. To describe this interface it is not enough to know the momenta and colors of all partons, we also have to know how they are connected. Each color charge must be associated with a partner anticharge. Therefore the interface is normally described by a string or a cluster chain. This intermediate interface is needed not only to determine the hadronization, it is also needed to specify the termination of the perturbative cascade. In the HERWIG [1,2] and ARIADNE [3] cascades the termination is specified by a local transverse momentum and in the PYTHIA/JETSET [4] cascade by the virtual mass of the parent. In all cases an ordering has to be known. We want to stress that in a perturbative calculation, which includes interference effects, the necessary parent-daughter relation cannot be specified from perturbative QCD. Instead it depends on the nonperturbative, soft, properties of the strong interaction. This implies that also if one assumes e.g. local parton hadron duality(LPHD), where the number of hadrons is directly related to the number of partons, a calculation of the multiplicity must rely on assumptions about the cutoff which cannot be specified from perturbative QCD alone. (These features have been more thoroughly discussed in Ref. [5].)

This problem does not appear in the large N_c limit. When $N_c \rightarrow \infty$ there is a single color ordering, which is the only possible one. Planar Feynman diagrams dominate and many interference terms vanish. In this limit we also find from perturbative QCD that generally partons are combined so as to produce a rather "short" string or cluster chain. The process $e^+e^- \rightarrow q\bar{q} + ng$ corresponds to a specific chain stretched by the gluons. In the perturbative phase, the only possibility to split the system into two or more separate subsystems, is by the process $g \rightarrow q\bar{q}$. Thus all strings or chains have a quark or an antiquark in the ends, and it is not possible to isolate a purely gluonic subsystem.

For finite N_c , there can be many different ways to connect the color charges. This ambiguity is connected with the possibility for two gluons to have identical colors, and is therefore generally of order $1/N_c^2$. New types of states are possible compared to the large N_c limit. Thus for the e^+e^- annihilation process it is possible to have a purely gluonic

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subsystem, which can form a color singlet and therefore hadronize as separated from the remainder. (This possibility was to our knowledge first discussed in Ref. [6].)

We are then facing the question: what decides the hadronization in a situation when several different color connections are possible:

- Does Nature make a random choice among the different possibilities? This means that the problem is determined purely by the $SU(3)$ group structure.
- Are some states dynamically favoured? If this is the case the probability for a specific configuration should depend on the combination of a group factor and a dynamic factor.
- If dynamics is important, are the properties in momentum space more essential, (e. g. favouring states which correspond to "short strings" Ref. [7]) or are the properties in coordinate space more important (as proposed e. g. in Ref. [8,9])?

We can isolate three different problems:

1. The $SU(3)$ group structure - How many possibilities are there, and what are their relative weights?
2. What are the dynamical properties of the alternative states allowed by the group structure?
3. What observables can be used to solve the problems?

In the standard models [1,10,11] for parton cascades and hadronization, the only effect of finite N_c originates from the coupling of a gluon to quarks, which is given by $C_F = \frac{1}{2}N_c(1 - 1/N_c^2)$ and thus reduced by a factor $(1 - 1/N_c^2)$ compared to the triple gluon coupling. Only contributions from planar Feynman diagrams are included and a set of color suppressed effects are neglected. This implies that in these models, just as in the large N_c limit, an $e^+e^- \rightarrow q\bar{q} + ng$ event can split into two separate strings or chains only via the process $g \rightarrow q\bar{q}$. Thus all strings or chains have a quark or an antiquark in the ends, and it is not possible to isolate a purely gluonic subsystem.

As mentioned above, the state intermediate between parton states and hadron states can be described by a string or cluster chain configuration. If this state contains a closed loop formed by gluons, we call it a color separate(CS) gluonic sub-singlet.

It is clear that to estimate the production rate of CS states, it is not enough to calculate the probability to find a gluonic subset which forms color singlet. For a state with many gluons this probability would always be one, and a gluonic singlet subsystem could also be formed in many different ways.

Due to their non-perturbative nature, there is no way so far to determine the probability of string states with CS gluonic sub-singlets. A set of models have been presented, which include nonconventional color configurations, where the partons can be "reconnected" compared to the "standard" configuration [8,9,12–14]. Such a model may easily appear as a black box in the sense that it is difficult to judge what property of the model is responsible for a particular observable effect. This paper is an attempt to make it easier to understand the features of CS states with gluonic sub-singlets. Here we use the method of the color effective Hamiltonian which is constructed from PQCD invariant amplitude [15] to study CS states corresponding to a particular string configuration from our available knowledge of PQCD.

In section II- IV, we study the effects of the group structure using this method. We try to estimate the probability to produce a color singlet with gluonic sub-singlets(CSGS), disregarding any dynamic effects which might disfavor its hadronization as a separate unit. For the parton states produced in PQCD the production amplitude can be written as a sum of terms, where each term contains a trace in color space, which corresponds to a definite color ordering of the gluons. We call these parton states $|h_i\rangle$. We also want to study a different set of states, called singlet chain states, for which the color of a gluon (or quark) and the anticolor of a neighbouring gluon (or antiquark) form a color singlet. These states can form a single chain (the states $|f_i\rangle$) or they can form two or more separate chains with at least one isolated gluonic loop (the states $|s_i\rangle$). In the large N_c limit the states $|h_i\rangle$ are identical to the states $|f_i\rangle$, and they correspond directly to string states with the same color ordering. For finite N_c this is no longer the case. It is not possible to directly associate the singlet chain states, $|f_i\rangle$ and $|s_i\rangle$, to definite string configurations. One reason is that the singlet chain states are not orthogonal. They can, however, as will be discussed in section IV, be adjusted into an orthogonal set by corrections of order $1/N_c^2$. We also show that to leading order in $1/N_c^2$ the number of possible states $|s_i\rangle$ is significantly reduced, which greatly simplifies the result. The states obtained in this way may be associated with specified string (or chain) configurations. Here we propose a first estimate for the group weight corresponding to a specific string configuration obtained by projecting these string candidates upon a specific $|h_i\rangle$.

In section V we present some results from a very simple "reconnection" model based on the rules about leading CSGSs which we obtain in the earlier section. The comparison between global properties of events with and without

CS states are given. By comparing with more elaborate models for e^+e^- annihilation one may hope to clarify if some simple regularities are most important for the result. The simple model might then be used to estimate reconnection effects in more complicated reactions.

II. COLOR EFFECTIVE HAMILTONIAN

The QCD Lagrangian is:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^\alpha F^{\alpha\mu\nu} + \bar{q}i\gamma^\mu \partial_\mu q + g\bar{q}\gamma^\mu A_\mu^\alpha T^\alpha q$$

where A_μ^α ($\alpha = 1, \dots, 8$) are eight gluon fields, q is the quark field with three colors (R, Y, B); field tensor $F_{\mu\nu}^\alpha$ is given by

$$F_{\mu\nu}^\alpha = \partial_\mu A_\nu^\alpha - \partial_\nu A_\mu^\alpha + gf^{\alpha\beta\gamma} A_\mu^\beta A_\nu^\gamma$$

and $T^\alpha = \lambda^\alpha/2$ where λ^α is the Gell-Mann matrix for $SU(3)$. We can write the QCD Lagrangian in another form by combining Gell-Mann matrix to a set of new ones:

$$\{\tau_{12} = \frac{1}{2}(\lambda_1 + i\lambda_2), \tau_{21} = \tau_{12}^\dagger, \tau_{13} = \frac{1}{2}(\lambda_4 + i\lambda_5), \tau_{31} = \tau_{13}^\dagger, \tau_{23} = \frac{1}{2}(\lambda_6 + i\lambda_7), \tau_{32} = \tau_{23}^\dagger, \lambda_3, \lambda_8\}$$

Matrix τ_{ij} raises color j to color i , and its Hermite conjugate $\tau_{ij}^\dagger = \tau_{ji}$ has the opposite function. Accordingly we define eight new gluon fields

$$X_\mu^3 = \frac{1}{\sqrt{2}}A_\mu^3, \quad X_\mu^8 = \frac{1}{\sqrt{2}}A_\mu^8$$

$$X_\mu^{21} = \frac{1}{\sqrt{2}}(A_\mu^1 + iA_\mu^2), \quad X_\mu^{12} = \frac{1}{\sqrt{2}}(A_\mu^1 - iA_\mu^2)$$

$$X_\mu^{31} = \frac{1}{\sqrt{2}}(A_\mu^4 + iA_\mu^5), \quad X_\mu^{13} = \frac{1}{\sqrt{2}}(A_\mu^4 - iA_\mu^5)$$

$$X_\mu^{32} = \frac{1}{\sqrt{2}}(A_\mu^6 + iA_\mu^7), \quad X_\mu^{23} = \frac{1}{\sqrt{2}}(A_\mu^6 - iA_\mu^7)$$

Hence the quark-gluon coupling part of the QCD Lagrangian can be written as:

$$\begin{aligned} \mathcal{L}_{int} &= g\bar{q}\gamma^\mu A_\mu^\alpha T^\alpha q \\ &= \frac{g}{\sqrt{2}}\bar{q}\gamma^\mu (\lambda^3 X_\mu^3 + \lambda^8 X_\mu^8) q \\ &\quad + \frac{g}{\sqrt{2}}[\bar{q}\gamma^\mu (\tau_{21} X_\mu^{21} + \tau_{31} X_\mu^{31} + \tau_{32} X_\mu^{32}) q + h.c.] \end{aligned} \quad (1)$$

where X_μ^3 couples to color $R\bar{R}$ and $Y\bar{Y}$; X_μ^8 to $R\bar{R}, Y\bar{Y}$ and $B\bar{B}$; X_μ^{21} to $R\bar{Y}$; X_μ^{12} to $Y\bar{R}$; and so on. So we write the QCD Lagrangian in a form with the quark-gluon interaction term showing clear color significance. This triggers our following color treatment for multigluon processes.

We can construct from PQCD a strict formulation to calculate the cross section of color singlets of a multiparton system at the tree level [15]. For the process $e^+e^- \rightarrow q\bar{q} + ng$, the essential part of the formulation is to exploit the color effective Hamiltonian H_c to compute the amplitude $\langle f | H_c | 0 \rangle$ of a certain color state $|f\rangle$. The color effective Hamiltonian H_c is found from the invariant amplitude $M_{ab}^{\alpha_1\alpha_2\cdots\alpha_n}$:

$$M_{ab}^{\alpha_1\alpha_2\cdots\alpha_n} = \sum_P (T^{\alpha_{P(1)}} T^{\alpha_{P(2)}} \cdots T^{\alpha_{P(n)}})_{ab} D^P \quad (2)$$

where a/b is the color index for quark/antiquark; $\alpha_1, \alpha_2, \dots, \alpha_n$ are those for n gluons with $\alpha_u = 1, \dots, 8$ (for $u = 1, \dots, n$); the summation is over all permutations of $(1, 2, \dots, n)$; $D^P \equiv D(q, \bar{q}, g_{P(1)}, g_{P(2)}, \dots, g_{P(n)})$ is the function of parton momenta where momentum indices are suppressed, and P denotes a certain permutation of $(1, 2, \dots, n)$;

$(T^{\alpha_{P(1)}} T^{\alpha_{P(2)}} \dots T^{\alpha_{P(n)}})_{ab}$ is the a -th row and the b -th column element of the matrix $(T^{\alpha_{P(1)}} T^{\alpha_{P(2)}} \dots T^{\alpha_{P(n)}})$. The form of the invariant amplitude can be derived from the $SU(3)$ color structure of QCD. A modern and convenient treatment was developed by Berends and Giele [16]. They proposed a recursive method from which eq.(2) can be readily obtained after including all Feynman diagrams at the tree level. Then H_c is built from eq.(2) as follows:

$$H_c = \sum_P (T^{\alpha_{P(1)}} T^{\alpha_{P(2)}} \dots T^{\alpha_{P(n)}})_{ab} D^P \Psi_a^\dagger \Psi_b^\dagger A_1^{\alpha_1 \dagger} A_2^{\alpha_2 \dagger} \dots A_n^{\alpha_n \dagger} \\ = \sum_P (1/\sqrt{2})^n Tr(Q^\dagger G_{P(1)}^\dagger G_{P(2)}^\dagger \dots G_{P(n)}^\dagger) D^P \quad (3)$$

where the repetition of two subscripts represents summation (we use this convention unless explicitly specified); $\Psi_a^\dagger = (R^\dagger, Y^\dagger, B^\dagger)$ is the color creation operator for quark; $\Psi^{b\dagger} = (\bar{R}^\dagger, \bar{Y}^\dagger, \bar{B}^\dagger)$ is the anticolor creation operator for antiquark; $(Q^\dagger)_a^b = \Psi^{b\dagger} \Psi_a^\dagger$ is the nonet tensor operator; For gluon u , G_u^\dagger , the gluon's color octet operator, is defined by:

$$G_u^\dagger = (1/\sqrt{2}) \lambda^{\alpha_u} A_u^{\alpha_u \dagger} = \Psi_u^{b\dagger} \Psi_{ua}^\dagger - \frac{1}{3} E \delta(a, b) \Psi_u^{b\dagger} \Psi_{ua}^\dagger \equiv G_u'^\dagger - \frac{1}{3} E \cdot 1_u \quad (4)$$

where $G_u'^\dagger \equiv \Psi_u^{b\dagger} \Psi_{ua}^\dagger$, $1_u \equiv \delta(a, b) \Psi_u^{b\dagger} \Psi_{ua}^\dagger$; $\Psi_u^{b\dagger}$ and Ψ_{ua}^\dagger are $(\bar{R}^\dagger, \bar{Y}^\dagger, \bar{B}^\dagger)_u$ and $(R^\dagger, Y^\dagger, B^\dagger)_u$ respectively; E is the unit 3×3 matrix; $\delta(a, b)$ is Kronecker symbol: $\delta(a, b) = \begin{cases} 1, & a = b \\ 0, & a \neq b \end{cases}$; $A_u^{\alpha_u \dagger}$ is the α_u -th gluon color operator whose meaning resembles that of X_μ field. Note that we use the same symbol A to denote these color operators as the gluon field in the former paragraph and this does not cause ambiguity because we no longer use the gluon field in the rest of the paper.

The color effective Hamiltonian is another expression of the S matrix, so it is not Hermitian. The validity of H_c can be verified by the calculation of the matrix element for the process $e^+ e^- \rightarrow q\bar{q} + ng$. For the color initial state $|0\rangle$ and the final state $|f\rangle = |\Psi_{a'} \Psi_{b'} A_1^{\alpha_1'} A_2^{\alpha_2'} \dots A_n^{\alpha_n'}\rangle$, summing over the color indices a'/b' for the quark/antiquark and those $\alpha_1', \alpha_2', \dots, \alpha_n'$ for n gluons, and carrying out all phase space integrals, we obtain:

$$\int d\Omega \sum_f |\langle f | H_c | 0 \rangle|^2 = \int d\Omega \langle 0 | H_c^\dagger H_c | 0 \rangle \\ = \int d\Omega M_{ab}^{\alpha_1 \alpha_2 \dots \alpha_n} \cdot (M_{ab}^{\alpha_1 \alpha_2 \dots \alpha_n})^* \\ = \sigma_{tree}(e^+ e^- \rightarrow q\bar{q} + ng) \quad (5)$$

where $\int d\Omega$ denotes phase space integrals including all kinematic factors. We can see that the calculation of the total cross section via H_c returns to the ordinary form.

A color state of the multiparton system $q\bar{q} + ng$ is composed of color charges of q, \bar{q} and n gluons. It belongs to the color space:

$$\underline{\mathfrak{z}}_q \otimes \underline{\mathfrak{z}}_{\bar{q}}^* \otimes \underline{\mathfrak{z}}_1 \otimes \underline{\mathfrak{z}}_2 \otimes \dots \otimes \underline{\mathfrak{z}}_n \quad (6)$$

or a little larger space:

$$\underline{\mathfrak{z}}_q \otimes \underline{\mathfrak{z}}_{\bar{q}}^* \otimes \underline{\mathfrak{z}}_1 \otimes \underline{\mathfrak{z}}_1^* \otimes \underline{\mathfrak{z}}_2 \otimes \underline{\mathfrak{z}}_2^* \otimes \dots \otimes \underline{\mathfrak{z}}_n \otimes \underline{\mathfrak{z}}_n^* \quad (7)$$

The effects of unphysical "singlet-gluon" state brought by enlarging the color space $\underline{\mathfrak{z}}_n$ to $\underline{\mathfrak{z}}_n \otimes \underline{\mathfrak{z}}_n^* (= \underline{\mathfrak{z}}_n \oplus \underline{\mathfrak{z}}_n)$ in (7) can be eliminated in calculating the matrix element of H_c , i.e. projecting the state to $H_c |0\rangle$, because

$$\langle 1_u \cdot \text{the rest of the color state} | H_c | 0 \rangle = 0$$

where u denotes a gluon. In other words, $\langle f | H_c | 0 \rangle = 0$ if $|f\rangle$ contains the singlet composed of the color and its anticolor of a single gluon. There are many ways of reducing the color space (6) or (7). If a complete and orthogonal singlet set is denoted by $\{|f_k\rangle, k = 1, 2, \dots\}$, we have:

$$|f_k\rangle \langle f_k| = 1 \quad (\text{completeness}), \quad \langle f_k | f_{k'} \rangle = \delta_{kk'} \quad (\text{orthogonality}) \quad (8)$$

and

$$\sum_k \sigma_k = \int d\Omega \sum_k |\langle f_k | H_c | 0 \rangle|^2 = \int d\Omega \langle 0 | H_c^\dagger | f_k \rangle \langle f_k | H_c | 0 \rangle \\ = \int d\Omega M_{ab}^{\alpha_1 \alpha_2 \dots \alpha_n} \cdot (M_{ab}^{\alpha_1 \alpha_2 \dots \alpha_n})^* = \sigma_{tree}(e^+ e^- \rightarrow q\bar{q} + ng) \quad (9)$$

This property implies that the sum of the cross sections over all color singlets in a complete and orthogonal singlet set for the system $(q\bar{q} + ng)$ is equal to the total cross section at the tree level.

III. EXAMPLES OF COMPLETE AND ORTHOGONAL SINGLET SETS

$$\text{A. } e^+e^- \rightarrow q\bar{q}g$$

For the process with only one gluon in the final state, i.e. $e^+e^- \rightarrow q\bar{q}g_1$, the color space is:

$$\underline{3}_q \otimes \underline{3}_{\bar{q}}^* \otimes \underline{8}_1 \quad (10)$$

The enlarged color space is:

$$\underline{3}_q \otimes \underline{3}_{\bar{q}}^* \otimes \underline{3}_1 \otimes \underline{3}_1^* \quad (11)$$

There is only one way of reducing the space (10). It corresponds to a unique singlet of the system $q\bar{q}g$. This singlet is $|Tr(QG_1)\rangle$, where $(Q)_a^b$ is the color nonet tensor of the quark pair and can be written as the irreducible octet tensor $(Q')_a^b$ plus its trace $1_Q \equiv Tr(Q)$: $Q = Q' + \frac{1}{3}1_Q E$. Thus the complete singlet set is composed of only one singlet:

$$\{|Tr(QG_1)\rangle\} = \{|Tr(Q'G_1)\rangle\} \quad (12)$$

where the equality is due to the fact that the gluon octet tensor is traceless, i.e. $Tr(G_1) = 0$.

There are many ways of reducing the space (11). Here we choose the following one that can lead to the singlet chain state:

$$(\underline{3}_q \otimes \underline{3}_1^*) \otimes (\underline{3}_{\bar{q}}^* \otimes \underline{3}_1) = (\underline{1}_{q1} \oplus \underline{8}_{q1}) \otimes (\underline{1}_{1\bar{q}} \oplus \underline{8}_{1\bar{q}}) \quad (13)$$

Corresponding to this reduction is the complete and orthogonal singlet set as follows:

$$\left\{ \frac{1}{3} |1_{q1} 1_{1\bar{q}}\rangle, \frac{1}{\sqrt{8}} |8_{q1} \otimes 8_{1\bar{q}}\rangle \right\} \quad (14)$$

where $1_{uv} \equiv \delta(a_u, b_v) \Psi_{a_u} \Psi_{b_v}^*$, and u and v denote the parton, a_u/b_v is the color/anticolor index for parton u/v ; and

$$|8_{uv} \otimes 8_{st}\rangle \equiv Tr(G_{uv}^\dagger G_{st}^\dagger) |0\rangle = |Tr(G_{uv} G_{st})\rangle$$

where u, v, s, t are parton labels and $(G_{uv}^\dagger)_a^b \equiv \Psi_v^{b\dagger} \Psi_u^a - \frac{1}{3}E\delta(a, b)\Psi_v^{b\dagger} \Psi_u^a$. The first singlet of (14) is defined as the singlet chain state. Generally a singlet chain state starts from the quark end, connects n gluons one by one in an order and ends at the antiquark end. Each piece of the chain state is a color singlet formed by the color of one parton and the anticolor of the next parton in this order. The second singlet of (14) is the orthogonal one of the singlet chain state and is obtained from the reduction $\underline{8}_{q1} \otimes \underline{8}_{1\bar{q}}$.

$$\text{B. } e^+e^- \rightarrow q\bar{q}gg$$

The color space is:

$$\underline{3}_q \otimes \underline{3}_{\bar{q}}^* \otimes \underline{8}_1 \otimes \underline{8}_2 \quad (15)$$

The enlarged color space is:

$$\underline{3}_q \otimes \underline{3}_{\bar{q}}^* \otimes \underline{3}_1 \otimes \underline{3}_1^* \otimes \underline{3}_2 \otimes \underline{3}_2^* \quad (16)$$

Let us look at one reduction way for space (15):

$$(\underline{3}_q \otimes \underline{3}_{\bar{q}}^*) \otimes (\underline{8}_1 \otimes \underline{8}_2) = (\underline{1}_{q\bar{q}} \oplus \underline{8}_{q\bar{q}}) \otimes (\underline{8}_1 \otimes \underline{8}_2) \quad (17)$$

The complete and orthogonal singlet set corresponding to this reduction is:

$$\{(2\sqrt{6})^{-1} |1_{q\bar{q}}(8_1 \otimes 8_2)\rangle, (3/80)^{1/2} |8_{q\bar{q}} \otimes \{8_1, 8_2\}\rangle, (48)^{-1/2} |8_{q\bar{q}} \otimes [8_1, 8_2]\rangle\} \quad (18)$$

where the symmetric octet state $|\{8_1, 8_2\}\rangle$ and the anti-symmetric one $|[8_1, 8_2]\rangle$ are defined as:

$$|\{8_1, 8_2\}\rangle = \left| \delta(a_1, b_2)(G_1)_{a_1}^b (G_2)_a^{b_2} + \delta(a_1, b_2)(G_2)_{b_2}^b (G_1)_a^{a_1} - \frac{2}{3}\delta(a, b)Tr(G_1 G_2) \right\rangle$$

$$|[8_1, 8_2]\rangle = |\delta(a_1, b_2)(G_1)_{a_1}^b (G_2)_a^{b_2} - \delta(a_2, b_1)(G_2)_{a_2}^b (G_1)_a^{b_1}\rangle$$

Another example is the reduction of the enlarged space (16) which can give the singlet chain state:

$$\begin{aligned} & (\underline{3}_q \otimes \underline{3}_1^*) \otimes (\underline{3}_1 \otimes \underline{3}_2^*) \otimes (\underline{3}_2 \otimes \underline{3}_{\bar{q}}^*) \\ &= (\underline{1}_{q1} \oplus \underline{8}_{q1}) \otimes (\underline{1}_{12} \oplus \underline{8}_{12}) \otimes (\underline{1}_{2\bar{q}} \oplus \underline{8}_{2\bar{q}}) \end{aligned} \quad (19)$$

The complete and orthogonal singlet set corresponding to this reduction is:

$$\begin{aligned} & \{(3\sqrt{3})^{-1} |1_{q1} 1_{12} 1_{2\bar{q}}\rangle, (2\sqrt{6})^{-1} |1_{q1} (8_{12} \otimes 8_{2\bar{q}})\rangle, (2\sqrt{6})^{-1} |1_{12} (8_{q1} \otimes 8_{2\bar{q}})\rangle, \\ & (2\sqrt{6})^{-1} |1_{2\bar{q}} (8_{q1} \otimes 8_{12})\rangle, (3/80)^{1/2} |8_{2\bar{q}} \otimes \{8_{q1}, 8_{12}\}\rangle, (48)^{-1/2} |8_{2\bar{q}} \otimes [8_{q1}, 8_{12}]\rangle\} \end{aligned} \quad (20)$$

In (20), the first singlet is the singlet chain state which has the parton configuration: $(qg_1g_2\bar{q})$. One can interchange gluon labels 1 and 2 in (19) to obtain another singlet set which includes the configuration $(qg_2g_1\bar{q})$.

IV. CSGS IN $Q\bar{Q} + NG$ SYSTEM

As we see in the second section, the color effective Hamiltonian H_c contains all perturbative dynamics relevant to color interaction. H_c is actually the color S matrix. We may define the $|in\rangle$ state as we do in quantum scattering theory:

$$|in\rangle \equiv |H_c\rangle \equiv H_c |0\rangle \quad (21)$$

We call the state $|H_c\rangle$ the physical color state. We can calculate the color matrix element of any color state $|f\rangle$ by projecting the state to the physical one: $\langle f | H_c \rangle$. According to (3), the physical state is written as:

$$|H_c\rangle = \sum_P (1/\sqrt{2})^n D^P |Tr(QG_{P(1)}G_{P(2)} \cdots G_{P(n)})\rangle \quad (22)$$

where there are $n!$ trace terms. Each of them is associated with one order of gluons: $P(1, 2, \dots, n-1, n)$, and it corresponds to a string connection of the same order at the large N_c limit because they are orthogonal to each other. But with $N_c = 3$, different trace terms overlap and no exact correspondence exists between a trace term with a specific gluon order in the physical color state at the partonic level and the string connection of the same order at the hadronic one. For each trace term, the color configuration is not uniquely determined and there are ingredients of color singlet chain states and CSGSs. In this section, we will discuss some properties of the CSGS and show how to estimate their probability from PQCD.

As the first example, let us look at the simplest case: $e^+e^- \rightarrow q\bar{q}g_1g_2$. The physical color state for this process is:

$$|H_c\rangle = \frac{1}{2} |Tr(QG_1G_2)\rangle D^{12} + \frac{1}{2} |Tr(QG_2G_1)\rangle D^{21} \quad (23)$$

According (18), the only CSGS for the system $q\bar{q}g_1g_2$ is:

$$\begin{aligned} |s_1\rangle &= \frac{1}{\sqrt{N_c}} \frac{1}{\sqrt{N_c^2-1}} |Tr(G_1G_2)1_{q\bar{q}}\rangle \\ &= \frac{1}{\sqrt{N_c}} \frac{1}{\sqrt{N_c^2-1}} (|1_{12}1_{21}1_{q\bar{q}}\rangle - \frac{1}{N_c} |1_11_21_{q\bar{q}}\rangle) \end{aligned} \quad (24)$$

where $\frac{1}{\sqrt{N_c}} \frac{1}{\sqrt{N_c^2-1}}$ is the normalization factor. The color part of the first trace term in the physical state is denoted by $|H_1\rangle$:

$$\begin{aligned} |H_1\rangle &= |Tr(QG_1G_2)\rangle \\ &= |1_{q1}1_{12}1_{2\bar{q}}\rangle - \frac{1}{N_c} |1_11_{q2}1_{2\bar{q}}\rangle - \frac{1}{N_c} |1_21_{q1}1_{1\bar{q}}\rangle + \frac{1}{N_c} |1_11_21_{q\bar{q}}\rangle \end{aligned} \quad (25)$$

where $|H_1\rangle$ has the order (g_1g_2) . We can make the state $|H_1\rangle$ normalized:

$$|h_1\rangle \equiv \frac{1}{\sqrt{N_c^3 - 2N_c + 1/N_c}} |H_1\rangle \quad (26)$$

From now on, for the sake of convenience, we will simply call the color part of a trace term in the full physical state (23) a trace state. So the projection of the state $|s_1\rangle$ on the normalized trace state $|h_1\rangle$ is:

$$\langle s_1 | h_1 \rangle \simeq \frac{1}{N_c} \quad (27)$$

We should compare the above result with that of two singlet chain states $|f_1\rangle = \frac{1}{\sqrt{N_c^3}} |1_{q1}1_{12}1_{2\bar{q}}\rangle$ and $|f_2\rangle = \frac{1}{\sqrt{N_c^3}} |1_{q2}1_{21}1_{1\bar{q}}\rangle$ which has the order $(qg_1g_2\bar{q})$ and $(qg_2g_1\bar{q})$ respectively. Their projections on the normalized trace state $|h_1\rangle$ are given by:

$$\begin{aligned} \langle f_1 | h_1 \rangle &= 1 \\ \langle f_2 | h_1 \rangle &= \frac{1}{N_c^2} \end{aligned} \quad (28)$$

Hence we conclude that the CSGS accounts for about $1/N_c^2$ of the trace state $|h_1\rangle$ and there is a small portion $(1/N_c^4)$ of $|f_2\rangle$ in $|h_1\rangle$ due to the different gluon order between the two states.

The projection of $|s_1\rangle$ on another trace term $|H_2\rangle = |Tr(QG_2G_1)\rangle$ is the same as on $|H_1\rangle$:

$$\langle s_1 | H_1 \rangle = \langle s_1 | H_2 \rangle \quad (29)$$

since there is no order for two gluons in $|H_1\rangle$ and $|H_2\rangle$: $|Tr(G_1G_2)\rangle = |Tr(G_2G_1)\rangle$. So the projection of $|s_1\rangle$ on the full physical state $|H_c\rangle$ is given by:

$$\langle s_1 | H_c \rangle = \frac{\sqrt{N_c - 1/N_c}}{2} (D^{12} + D^{21}) \quad (30)$$

The probability for the parton system $q\bar{q}g_1g_2$ to be in $|s_1\rangle$ is obtained by:

$$\begin{aligned} Prob(|s_1\rangle) &= \frac{1}{\sigma_0} \int d\Omega |\langle s_1 | H_c \rangle|^2 \\ &= \frac{1}{\sigma_0} \frac{N_c^2 - 1}{4N_c} \int d\Omega (|D^{12}|^2 + |D^{21}|^2 + 2Re(D^{12}D^{21*})) \end{aligned} \quad (31)$$

where $\sigma_0 \equiv \sigma_{tree}(e^+e^- \rightarrow q\bar{q}g_1g_2)$ is the tree level cross section for $e^+e^- \rightarrow q\bar{q}g_1g_2$.

There is only one CSGS for $q\bar{q}g_1g_2$. As the number of gluons increases, more such states arise. For the second example, we investigate the process: $e^+e^- \rightarrow q\bar{q}g_1g_2g_3$. The full physical color state for the process is:

$$\begin{aligned} |H_c\rangle &= \frac{1}{(\sqrt{2})^3} [D^{123} |Tr(QG_1G_2G_3)\rangle + D^{132} |Tr(QG_1G_3G_2)\rangle + D^{213} |Tr(QG_2G_1G_3)\rangle \\ &+ D^{231} |Tr(QG_2G_3G_1)\rangle + D^{312} |Tr(QG_3G_1G_2)\rangle + D^{321} |Tr(QG_3G_2G_1)\rangle] \end{aligned} \quad (32)$$

Without loss of generality, we choose one trace term, e.g. $|H_1\rangle \equiv |Tr(QG_1G_2G_3)\rangle$ to go through our discussion. The other situations can be instantly obtained by permuting the gluon labels.

As in the case of $e^+e^- \rightarrow q\bar{q}g_1g_2$, when N_c is large, the normalized state $|h_1\rangle$ is approximately the same as that of the singlet chain state $|f_1\rangle \equiv (1/N_c^2) |1_{q1}1_{12}1_{23}1_{3\bar{q}}\rangle$, noting that $|f_1\rangle$ has the same order of gluons as $|h_1\rangle$. There are five CSGSs for the system $q\bar{q}g_1g_2g_3$:

$$\begin{aligned} &\{|s_i\rangle, i = 1, 2, 3, 4, 5\} \\ &= \{A |Tr(G_1G_2)1_{q3}1_{3\bar{q}}\rangle, A |Tr(G_2G_3)1_{q1}1_{1\bar{q}}\rangle, A |Tr(G_1G_3)1_{q2}1_{2\bar{q}}\rangle, \\ &B |Tr(G_1G_2G_3)1_{q\bar{q}}\rangle, B |Tr(G_1G_3G_2)1_{q\bar{q}}\rangle\} \end{aligned} \quad (33)$$

where A and B are normalization factors and given by:

$$A = \frac{1}{N_c \sqrt{N_c^2 - 1}}, \quad B = \frac{1}{\sqrt{N_c^4 - 3N_c^2 + 2}} \quad (34)$$

These singlets are all made up of two sub-singlets. For the first three states($i=1,2,3$), one sub-singlet is formed by two gluons, the other is by the remaining gluon with the quark-antiquark pair, while for the last two states($i=4,5$), all three gluons form one sub-singlet and the quark-antiquark pair forms the other one. Their projections on the normalized trace state $|h_1\rangle$ are:

$$\begin{aligned}
|\langle s_1 | h_1 \rangle|^2 &= |\langle s_2 | h_1 \rangle|^2 \simeq 1/N_c^2 \\
|\langle s_3 | h_1 \rangle|^2 &\simeq 1/N_c^6 \\
|\langle s_4 | h_1 \rangle|^2 &\simeq 1/N_c^2 \\
|\langle s_5 | h_1 \rangle|^2 &\simeq 1/N_c^6
\end{aligned} \tag{35}$$

We see that the projection squares of $|s_1\rangle$, $|s_2\rangle$ and $|s_4\rangle$ on $|h_1\rangle$ are all about $1/N_c^2$ to the leading order in N_c , while those of $|s_3\rangle$ and $|s_5\rangle$ are suppressed by an additional factor $1/N_c^4$. Recalling (33), we may write down the parton orders for all these CSGSs below:

$$\begin{aligned}
(12)(03) &\rightarrow |s_1\rangle \\
(01)(23) &\rightarrow |s_2\rangle \\
(02)(13) &\rightarrow |s_3\rangle \\
(0)(123) &\rightarrow |s_4\rangle \\
(0)(132) &\rightarrow |s_5\rangle
\end{aligned} \tag{36}$$

where a parenthesis denotes a sub-singlet; 0 stands for the quark-antiquark pair and 1,2,3 for gluons. We compare these orders with the order in $|H_1\rangle$: (0123). For $|s_1\rangle$, the orders in both sub-singlets are the same as (0123), while for, e.g. $|s_3\rangle$, the order in both sub-singlets are different from (0123) because both 0 and 1 are not adjacent to 2 and 3 in (0123) respectively. We conclude that the parton orders in $|s_1\rangle$, $|s_2\rangle$ and $|s_4\rangle$ have the least differences from (0123), while those in $|s_3\rangle$ and $|s_5\rangle$ are more different from (0123) than the former three states. In other words, each sub-singlet of $|s_1\rangle$, $|s_2\rangle$ and $|s_4\rangle$ conserves the parton order in the trace state at most, so they have the largest overlap with the trace state. The reason that $|s_3\rangle$ and $|s_5\rangle$ are more suppressed is that their sub-singlets do not all keep the parton order in the trace state. The inner product of any two states in $|s_1\rangle$, $|s_2\rangle$ and $|s_4\rangle$ is about $1/N_c^2$, while that of one state in $|s_1\rangle$, $|s_2\rangle$ and $|s_4\rangle$ with one of $|s_3\rangle$ and $|s_5\rangle$ is about $1/N_c^3$ or higher.

This rule can be further verified for $e^+e^- \rightarrow q\bar{q}g_1g_2g_3g_4$. CSGSs with the leading contribution to $|H_1\rangle = |Tr(QG_1G_2G_3G_4)\rangle$ are:

$$\{(123)(04), (01)(234), (12)(034), (23)(014), (012)(34), (0)(1234)\} \tag{37}$$

Their projection squares on the normalized trace state $|h_1\rangle$ are all $1/N_c^2$ to the leading order in $1/N_c$. The inner product of any two different states in the above list is about $1/N_c^2$. Note that we only include states with two sub-singlets because the projection squares on $|h_1\rangle$ for those with three or more sub-singlets are at least suppressed by additional $1/N_c^2$ as compared to those with only two.

For cases of n -gluons where $n \leq 6$, the above conclusions can be verified by direct calculation. For cases of $n > 6$, a formal proof is given below. Suppose any two different leading CSGSs have an inner product of $1/N_c^2$ to the leading order in $1/N_c$ in n -gluons case. Let us prove that the same statement holds for $(n+1)$ -gluons case. We focus on two different CSGSs which we call A and B:

$$\begin{aligned}
A : (I)(II) &= (0, 1, 2, \dots, k_1, k_1 + l_1 + 1, \dots, n+1)(k_1 + 1, k_1 + 2, \dots, k_1 + l_1) \\
B : (III)(IV) &= (0, 1, 2, \dots, k_2, k_2 + l_2 + 1, \dots, n+1)(k_2 + 1, k_2 + 2, \dots, k_2 + l_2)
\end{aligned} \tag{38}$$

where 0 stands for $q\bar{q}$ and the parentheses for sub-singlets. Each CSGS has a gluonic sub-singlet with l_1 and l_2 gluons, respectively. For $n > 6$, at least a common piece of $[a_1, a_2, a_3]$ in two CSGSs must exist. Here a_1 , a_2 and a_3 denote three consecutive gluons in a sub-singlet of one CSGS. When we calculate the inner product of A and B, we get a factor of N_c^2 to the leading order in N_c from this common piece. Corresponding to A and B in $(n+1)$ -gluons case, there exist two CSGSs A' and B' in n -gluons case where all parts of A'/B' are the same as A/B except that the common piece $[a_1, a_2, a_3]$ is changed to $[a_1, a_2]$ (a_3 as the new added gluon to A and B) or $[a_2, a_3]$ (a_1 as the new added gluon to A and B) or $[a_1, a_3]$ (a_2 as the new added gluon to A and B). Note that any one of three cases are equivalent to our proof. The contribution from the common piece, say $[a_1, a_2]$, to the inner product of A' and B' is N_c to the leading order in $1/N_c$, while that from the common piece $[a_1, a_2, a_3]$ to the inner product of A and B is about N_c^2 . The normalization factor of about $N_c^{-(n+2)}$ for $(n+1)$ -gluons case and $N_c^{-(n+1)}$ for n -gluons one must also be taken into account. Considering contributions from both the common piece and the normalization factor, we find that the inner product of A and B is the same as that of A' and B' to the leading order in $1/N_c$. The same strategy can be also used to prove that the inner product between any leading CSGS and its corresponding trace state is about $1/N_c$.

Hence we summarize the following rules for the CSGS with the leading contribution to a trace state:

- The state has only two sub-singlets to the leading order in $1/N_c$. Its projection square on the trace state is about $1/N_c^2$.
- Each sub-singlet keeps its parton order as that in the trace state.

- The inner product of any two different leading contribution states is about $1/N_c^2$, while that of a leading state with a non-leading one is about $1/N_c^3$ or higher.

According to the above rules, we can determine for the process $e^+e^- \rightarrow q\bar{q}g_1g_2\cdots g_{n-1}g_n$ the total number of CSGSs with the leading contribution to a trace state. We suppose this trace state corresponds to the trace term $|Tr(QG_1G_2G_3\cdots G_{n-1}G_n)\rangle$. Hence, for the configuration $(q\bar{q}g_1g_2\cdots g_{n-1}g_n)$, there are $(n-1)$ states with one sub-singlet formed by 2 gluons and the other by the remaining gluons with the quark-antiquark pair. Obviously, there are $(n-k+1)$ states with one k -gluon sub-singlet and the other formed by the remaining partons. Note that all these sub-singlets keep their parton orders as that in the trace state. Altogether, the total number is:

$$1 + 2 + 3 + \cdots + n - 1 = \frac{1}{2}n(n-1) \quad (39)$$

We hereafter denote this number by $m = \frac{1}{2}n(n-1)$.

Now we start to discuss how to estimate the total probability of the CSGS. One problem we are facing is that CSGSs are not orthogonal to each other. One cannot obtain the total probability by summing all individual probability values for each state. The non-orthogonality would make the total probability derived in this way different from its correct value because the contribution from the overlap of any two states would be counted multiply. So for non-orthogonal states, the orthogonalization is needed. Let us illustrate the procedure more explicitly. If we have a set of non-orthogonal states:

$$\{|f_i\rangle, i = 1, 2, \cdots, l\} \quad (40)$$

suppose we have finished the orthogonalization and find a set of orthogonal states as follows:

$$\{|f'_i\rangle, i = 1, 2, \cdots, l\} \quad (41)$$

The total probability for the set of states is:

$$\begin{aligned} Prob(\sum |f'_i\rangle) &= \frac{1}{\sigma_0} \sum_i \int d\Omega |\langle f'_i | H_c \rangle|^2 \\ &= \frac{1}{\sigma_0} \frac{1}{2^n} \sum_i \int d\Omega \left| \sum_P D^P \langle f'_i | Tr(QG_{P(1)}G_{P(2)}\cdots G_{P(n)}) \rangle \right|^2 \end{aligned} \quad (42)$$

where $\sigma_0 \equiv \sigma_{tree}(e^+e^- \rightarrow q\bar{q}g_1g_2\cdots g_n)$ is the tree level cross section for $e^+e^- \rightarrow q\bar{q}g_1g_2\cdots g_n$.

As stated above, it is necessary to orthogonalize the non-orthogonal CSGSs to give their total probability. But actually this job proves to be extremely difficult. The greatest obstacle lies in that the number of CSGSs (which is also the dimension of the transformation matrix from original states to orthogonal ones) is huge when the gluon number is large, and finding the transformation matrix with this huge dimension is almost impossible. All possible CSGSs must be taken into account, not only those with the leading contribution to a specific trace term of the full physical state $|H_c\rangle$. The reason for this is simple: $|H_c\rangle$ includes all trace terms each of which corresponds to one order of gluons, hence a non-leading singlet for one trace term is not necessarily still the non-leading one for other trace terms with different gluon orders. Another big obstacle is that calculating the momentum function D^P is a very complicated task, especially for multigluon processes. To some degree, these two difficulties make the orthogonalization of more theoretical value.

The orthogonalization is, however, really simple and heuristic in some cases. Let us look at such a case. Assume that the physical state is fixed at one trace term, e.g. $|H\rangle \equiv |Tr(QG_1G_2G_3\cdots G_{n-1}G_n)\rangle$, we now try to estimate the probability of the CSGS in this case. Before we orthogonalize these singlets, we make some further approximations:

- We only take into account the leading contribution singlets as determined by the rule on page 8. All the inner products of two different leading singlets are supposed to be the same $1/N_c^2$. In fact, they are the same to the leading order in $1/N_c$.
- There is freedom to choose the transformation matrix (the definition is given below). Considering the symmetry of inner products assumed in the previous item, we choose a simplest form for the transformation matrix: all the diagonal elements are assumed to be equal, and so are the non-diagonal elements.

Keeping these approximations in mind, we will proceed with the problem. Now we have m non-orthogonal states:

$$\{|s_i\rangle, i = 1, 2, \cdots, m\} \quad (43)$$

where the inner product of any two states is given by:

$$C_{ij} = C_{ji} = \langle s_i | s_j \rangle = \begin{cases} 1, & \text{for } i = j \\ 1/N_c^2, & \text{for } i \neq j \end{cases} \quad (44)$$

Our goal is to find m orthogonal states based on (43). These states are denoted by:

$$\{|s'_i\rangle, i = 1, 2, \dots, m\}$$

They are related to original states (43) by a linear transformation:

$$|s'_i\rangle = U_{ij} |s_j\rangle \quad (45)$$

where the transformation matrix U is a real $m \times m$ matrix. It is not the only transformation which transforms the original non-orthogonal states to orthogonal ones. Different transformation matrices correspond to different orthogonal sets. Two transformation matrices are connected through a unitary transformation, but the transformation matrix itself is not unitary because it associates orthogonal states with non-orthogonal ones. Any transformation matrix is equivalent to others in calculating the probability. The inner product of two new states is:

$$\langle s'_i | s'_j \rangle = \delta(i, j) = \sum_{k,l} U_{ik} U_{jl} \langle s_k | s_l \rangle = \sum_{k,l} U_{ik} U_{jl} C_{kl} \quad (46)$$

As we state in the above, from the symmetric property of C_{kl} in (44), we choose a symmetric matrix U . We search for a solution for which all diagonal elements have the same value and also all non-diagonal elements are equal to each other:

$$\begin{aligned} U_{ii} &= a \\ U_{ij} &= b, \quad \text{for } i \neq j \end{aligned} \quad (47)$$

where a and b are real numbers. Thus (46) becomes:

$$\begin{aligned} \sum_{k,l} U_{ik} U_{il} C_{kl} &= 1 \\ \sum_{k,l} U_{ik} U_{jl} C_{kl} &= 0, \quad \text{for } i \neq j \end{aligned} \quad (48)$$

Substituting (47) into (48), we obtain:

$$\begin{aligned} a^2 + (m-1)b^2 + \frac{2}{N_c^2}(m-1)ab + \frac{1}{N_c^2}(m-1)(m-2)b^2 &= 1 \\ 2ab + (m-2)b^2 + \frac{1}{N_c^2}a^2 + \frac{2}{N_c^2}(m-2)ab + \frac{1}{N_c^2}(m-2)(m-3)b^2 &= 0 \end{aligned} \quad (49)$$

We find the exact solution for (49):

$$a^2 = \frac{N_c^2}{(m^2 - 3m + 2 + (m-1)N_c^2)y^2 + 2(m-1)y + N_c^2}, \quad b = ya \quad (50)$$

where y is given by:

$$y = \frac{-(m + N_c^2 - 2) \pm \sqrt{(N_c^2 + 1)m + N_c^4 - 2N_c^2 - 2}}{(m-2)(m + N_c^2 - 3)} \quad (51)$$

We note that this solution is finite when $m \rightarrow 2$, and therefore regular for all positive values of m . We choose the solution with a positive sign, as this corresponds to a unit matrix, $U = 1$, in the limit $N_c \rightarrow \infty$. The orthogonal states are:

$$|s'_i\rangle = a |s_i\rangle + b \sum_{j \neq i} |s_j\rangle \quad (52)$$

The overlap between the trace state $|h\rangle$ and the states $|s_j\rangle$ is given by

$$\sum_i |\langle s'_i | h \rangle|^2 = \frac{m}{N_c^2} (a + (m-1)b)^2 \quad (53)$$

When N_c is large we find $y \simeq -\frac{1}{2N_c^2}$ and $a^2 \simeq 1$. Thus to order $1/N_c^2$ the probability behaves as $\frac{m}{N_c^2}$ which increases monotonously with increasing number of gluons. To estimate the probability we introduce the singlet chain state $|f\rangle$

into the list of states to be orthogonalised. Note that $|f\rangle$ has the same gluon order as the chosen trace state $|h\rangle$ to which the CSGS $\{|s_i\rangle\}$ or $\{|s'_i\rangle\}$ correspond. We define the new orthogonal states as follows:

$$\begin{aligned} |s''_i\rangle &= \alpha |s'_i\rangle + \beta |f\rangle \\ |f'\rangle &= \gamma |f\rangle + \delta \sum_k |s'_k\rangle \end{aligned} \quad (54)$$

When N_c is large, keeping only the leading terms in N_c , (52) becomes

$$|s'_i\rangle \simeq |s_i\rangle - \frac{1}{2N_c^2} \sum_{j \neq i} |s_j\rangle \quad (55)$$

which implies $\langle f | s'_i \rangle \simeq 1/N_c$. $\alpha, \beta, \gamma, \delta$ are four parameters to be determined from the following orthogonality:

$$\begin{aligned} \langle s''_i | s''_i \rangle &= 1 \\ \langle s''_i | s''_j \rangle &= 0 \quad \text{for } i \neq j \\ \langle f' | f' \rangle &= 1 \\ \langle f' | s''_i \rangle &= 0 \end{aligned} \quad (56)$$

From the first and second equation of (56), we obtain $\alpha = 1$ and $\beta = -2/N_c$. From the last two equations of (56), we get to order $1/N_c^2$

$$\gamma^2 \approx 1 - \frac{3m}{N_c^2}; \quad \delta \approx \frac{\gamma}{N_c} \quad (57)$$

In the large N_c limit, the squared projections of $|f'\rangle$ and $|s''_i\rangle$ on the corresponding normalized trace state $|h\rangle$, i.e. the probabilities of $|f'\rangle$ and $|s''_i\rangle$ in $|h\rangle$, are given by:

$$\begin{aligned} P_{f'} &\approx |\langle f' | h \rangle|^2 = \gamma^2 (1 + m/N_c^2)^2 \simeq \frac{1}{1 + \frac{m}{N_c^2}} \\ P_{s''} &= |\langle s''_i | h \rangle|^2 \approx 1/N_c^2 \simeq \frac{1}{N_c^2(1 + m/N_c^2)} \end{aligned} \quad (58)$$

In the last approximations we have added terms of higher order in $1/N_c^2$ in such a way that the total probability adds up to one for all values of m . Thus the total probability for all $|s''\rangle$ states becomes $mP_{s''} \simeq \frac{m/N_c^2}{1 + m/N_c^2}$, which increases monotonously with growing gluon number (or m). Correspondingly $P_{f'}$ decreases monotonously with increasing m .

Summarily, what we have derived gives us some valuable messages about the properties of the CSGS. Here are some of them: (a) The probability of color separate singlets cannot be neglected. They may account for a considerable fraction of the total events. We try to estimate the probability to produce a CSGS, disregarding any dynamic effects which might disfavor its hadronization as a separate unit. (b) The set of states corresponding to different string configurations must necessarily be orthogonal, as they correspond to different values for observables. In the large N_c limit, CSGSs do not appear. A trace state is just the singlet chain state having the same gluon order. All singlet chain states are orthogonal to CSGSs. For finite N_c , neither are the three sets of states, trace states, singlet chain states and color separate states, orthogonal among themselves, nor is a set of states orthogonal to any other. These states are not possible candidates to represent pure string states. Keeping only the first correction term in an expansion in $1/N_c^2$, it is, however, possible to construct modified orthogonal states which corresponds to a specific trace state $|h_i\rangle$: $\{f'_i, \{s''_{ij}\}\}$. Thus they form an orthogonal set of states, which may be associated with specified string (or chain) configurations. Here we propose a first estimate for the group weight corresponding to a specific string configuration obtained by projecting these string candidates upon a specific $|h_i\rangle$. (c) For a trace state, leading CSGSs obey the rules on page 8. In particular the rules are crucial for us to build a practical model to describe the production of color separate states. This is the subject which we will discuss in the next section.

V. A COLOR SEPARATE STATE MODEL

According to the arguments in the last section, the total percentage of color separate states should not be negligibly small, especially in multigluon process. It is of significance to introduce CS states in the current event generators to see how they influence the event shape and properties. In this section, we will build such a practical model based on and implementing the rules which we derived in the former section.

Firstly, the difference between two terms, the CSGS and CS state, should be clarified. We mention in the introduction that to describe the interface between the perturbative stable partonic state and the hadronic one, it is

not enough to know the momenta and colors of all partons, we also have to know how they are connected. Their connection can be described by a string or cluster chain configuration where each color charge must be associated with a partner anticharge. The connection or the string/chain ordering cannot be specified by perturbative QCD alone. It depends on the nonperturbative, soft, properties of the strong interaction. If the string or cluster chain state contains a closed loop formed by gluons we call it a CS gluonic sub-singlet which is a non-perturbative object, while a CSGS is defined by the colors of partons which is of perturbative nature. A CSGS is not necessarily a CS state. In this section, we use JETSET to implement the fragmentation which is a non-perturbative soft process. Hence we start using the term "CS state" in most places to describe the true objects formed at the end of the parton cascade or at the beginning of the hadronization.

The outline of the model is as follows.

1. We use a parameter R to describe the total percentage of CS states in all events. We use JETSET to produce a configuration of partons where for the singlet or for each sub-singlet of the type $qg_1g_2 \cdots g_l\bar{q}$, the color flow starts at the q , connecting gluons one by one and ends at \bar{q} . This color connection corresponds to a trace term in the physical color state $|H_c\rangle$ which we call a trace state.
2. We only consider those CSGSs with leading contributions to the trace state. For a configuration $qg_1g_2 \cdots g_l\bar{q}$, there are $m = \frac{1}{2}l(l-1)$ leading states according to (39). Among them there are $(l-1)$ CSGSs of 2-gluon type, $(l-2)$ states of 3-gluon type, ..., $(l-k+1)$ states of k -gluon type, etc.. Any two different CSGSs have approximately the same inner product ($\simeq 1/N_c^2$). Each state's square of projection on the trace state (corresponding to the current color configuration) is the same ($\simeq 1/N_c^2$). So all CSGSs play an equal role. However, not all of these states finally contribute to the event. Thanks to the fact that JETSET records the whole branching history for all partons, we can exclude those CSGSs where all gluons in the gluon-type sub-singlet are emitted from a single octet gluon. After filtered by this procedure, suppose there are m' leading CSGSs remaining.
3. We consider two kinds of weights to select a CS state corresponding to the given configuration. One is the constant weight, i.e. each CS state is selected with equal probability. This is a natural choice if we only consider the color part and neglect the momentum or phase space sector. The other is T(or λ) measure weight (we will come to this topic later in details). For each CS state, we have a T-weight. So there are m' T-weights corresponding to m' CS states. A CS state for the current configuration is selected according to these weights.
4. The fragmentation of the selected CS state is implemented by the Lund string model. The difference between an ordinary state of the type $qg_1g_2 \cdots g_l\bar{q}$ and a CS state is that the former state forms an open string while the latter forms a closed string and an open one.

In order to better understand our Monte Carlo simulation results, we should use a little more space for illustrating the physical meaning of the T-measure. We will address two subjects: (a) How is the T-measure defined and what's its physical meaning? (b) Its relationship to the λ -measure in the Dipole Cascade Model (DCM).

For a $q\bar{q}$ system the produced hadrons are evenly distributed in rapidity, which means that their energy-momentum four-vectors, when plotted one by one, are distributed around a hyperbola as seen in Fig.1a. For a $q\bar{q}g$ system, the two string pieces will produce hadrons such that the momentum four vectors lie around two hyperbolae (Fig.1b). This corresponds to three jets along the parton momenta, and the jets are smoothly connected to each other. For a multigluon system it is possible to generalize the hyperbolae in the $q\bar{q}$ and $q\bar{q}g$ cases and define a timelike curve in energy-momentum space. This curve (which we call the x-curve) follows the color ordered parton momenta in such a way that the corners are smoothed out with a resolution power given by a parameter m_0 . The directrix curve A_μ for the string parton state is obtained by placing the four-momenta of all the partons one by one according to their color ordering, see Fig.1c. When $m_0 \rightarrow 0$, the x-curve becomes the directrix. Let ξ be a parameter which runs along the directrix. To every point A_μ on the directrix there is a corresponding point x_μ on the x-curve. So the x-curve which is determined through the differential equations [19]:

$$\begin{aligned} dx_\mu(\xi) &= \frac{1}{m_0^2}(q \cdot dA)q_\mu \\ x_\mu(0) &= A_\mu(0) = 0 \end{aligned} \quad (59)$$

where $q_\mu(\xi) \equiv A_\mu(\xi) - x_\mu(\xi)$ is tangent to the x-curve. T is defined by:

$$\begin{aligned} d(\ln T) &= \frac{q \cdot dA}{m_0^2} = \frac{\sqrt{dx^2}}{\sqrt{q^2}} \equiv d\lambda \\ T &= \exp(\lambda) \end{aligned} \quad (60)$$

Note that T is just the exponential of the area spanned by the x-curve and the directrix. T can also be interpreted as the exponential of the invariant length of the x-curve which we call the λ -measure. The square amplitude to obtain

a certain string state with n partons is given by $\frac{1}{T_n(k_1, k_2, \dots, k_n)}$ where k_1, k_2, \dots, k_n are four-momenta of n partons. There is a recursive expression for T_n :

$$\begin{aligned} q_{j+1} &= \gamma_{j+1} q_j + \frac{1}{2}(1 + \gamma_{j+1}) k_{j+1} \\ \gamma_{j+1} &= 1/[1 + \frac{q_j \cdot k_{j+1}}{m_0^2}] \\ T_n^{-1} &= \gamma_2 \gamma_3 \cdots \gamma_n \end{aligned} \quad (61)$$

The T-measure defined as above depends on the initial value of q_μ . It is possible to start with the initial value $q_0 = 0$ for an open string. We regard the sequence k_1, k_2, \dots, k_n as the first half of a full directrix period, the second half is k_n, k_{n-1}, \dots, k_1 , which is in the opposite order. This is one period: $k_1, k_2, \dots, k_n, k_n, k_{n-1}, \dots, k_1$. After several periods of recursive calculation according to (61), we will get for a period a stable T which is insensitive to the initial value q_0 . For a closed string where there is no obvious beginning, as a choice, we also use this strategy to get a stable T .

In the string fragmentation model, the λ -measure is the effective rapidity range given by: $\lambda \sim \sum \ln[(k_i + k_{i+1})^2/m_0^2]$. Hence, according to the weight defined by $\frac{1}{T} = \exp(-\lambda)$, we may tell which string parton state is "short" and which is "long". Here a "short" string means the string with small λ . The weight implies that a short string parton state is more favored than a long one.

For a configuration $(qg_1 g_2 \cdots g_n \bar{q})$ produced by JETSET, suppose an allowed CS state is made up of a separate sub-singlet $(g_i g_{i+1} \cdots g_{i+j-1})$ of the j -gluon type and a sub-singlet $(qg_1 \cdots g_{i-1} g_{i+j} g_{i+j+1} \cdots g_n \bar{q})$ of the ordinary type. The total weight is:

$$\begin{aligned} \frac{1}{T} &= \frac{1}{T_1(k_i, k_{i+1}, \dots, k_{i+j-1}, k_i) T_2(k_q, k_1, \dots, k_{i-1}, k_{i+j}, k_{i+j+1}, \dots, k_n, k_{\bar{q}})} \\ &\equiv \exp(-\lambda_1 - \lambda_2) \end{aligned} \quad (62)$$

There is a difference between T_1 and T_2 : T_1 is the T-measure for a closed string, so it is written as:

$$T_1 = \gamma_{k_{i+1}} \gamma_{k_{i+2}} \cdots \gamma_{k_{i+j-1}} \gamma_{k_i} = \exp(-\lambda_1) \quad (63)$$

where the recursive sequence $(k_i, k_{i+1}, \dots, k_{i+j-1}, k_i)$ starts at k_i to k_{i+j-1} and then back to k_i ; and λ_1 is approximately:

$$\lambda_1 \sim \ln(s_{i,i+1}/m_0^2) + \ln(s_{i+1,i+2}/m_0^2) + \cdots + \ln(s_{i+j-2,i+j-1}/m_0^2) + \ln(s_{i+j-1,i}/m_0^2) \quad (64)$$

where $s_{ij} \equiv (k_i + k_j)^2$ and the last term is the additional term compared to an open string. There is no such a term for an open string formed by the same sequence of momenta. We also use the λ -measure for the full configuration $(qg_1 g_2 \cdots g_n \bar{q})$ without interruption by separate sub-singlets:

$$\lambda_0 \sim \ln(s_{1,2}/m_0^2) + \ln(s_{2,3}/m_0^2) + \cdots + \ln(s_{n-2,n-1}/m_0^2) + \ln(s_{n-1,n}/m_0^2) \quad (65)$$

We can compare λ_0 with $\lambda_1 + \lambda_2$ by investigating their difference $(\lambda_1 + \lambda_2) - \lambda_0$:

$$\begin{aligned} &\lambda_1 + \lambda_2 - \lambda_0 \\ &\sim \ln(s_{i+j-1,i}) + \ln(s_{i-1,i+j}) - \ln(s_{i-1,i}) - \ln(s_{i+j-1,i+j}) \\ &= \ln\left(\frac{(k_{i+j-1} \cdot k_i)(k_{i-1} \cdot k_{i+j})}{(k_{i-1} \cdot k_i)(k_{i+j-1} \cdot k_{i+j})}\right) \end{aligned} \quad (66)$$

where k_{i-1} and k_i , k_{i+j-1} and k_{i+j} are momenta of two gluons in neighborhood in the color flow respectively, while k_{i+j-1} and k_i , k_{i-1} and k_{i+j} are not adjacent. Normally, we have:

$$(k_{i+j-1} \cdot k_i)(k_{i-1} \cdot k_{i+j}) > (k_{i-1} \cdot k_i)(k_{i+j-1} \cdot k_{i+j}) \quad (67)$$

which implies $\lambda_1 + \lambda_2 > \lambda_0$. That is to say: the length of a CS state is usually longer than the ordinary state without interruption. We expect that the shorter the length of a string state is, the more favored it would be. We can find some configurations for a CS state where its length $\lambda_1 + \lambda_2$ is shorter. It is conceivable that these configurations are more favored and have larger production weights. For example, if all the j gluons in the separate sub-singlet plus two neighbor gluons to its two ends are soft, then the weight must be large because separating the j -gluon as a sub-singlet will not change the original λ_0 very much.

To see the situation more easily and intuitively, we resort to the λ diagram which is often used in the DCM. Fig.2 is the diagram of the effective rapidity range (i.e. λ) for a parton configuration [20]. In this configuration, there are nine gluons. The order along the color flow is shown in the figure which reads $qg_1 g_2 \cdots g_9 \bar{q}$. The length of the

folded line $q123456789\bar{q}$ is λ_0 . The length of the folded line between two neighboring gluons, e.g. i and $i+1$, is $\ln(s_{i,i+1}/m_0^2)$. In the figure, there are 11 ends labeled by q, \bar{q} and nine gluons respectively. We call these ends twigs. Along the line $q\bar{q}$, there are 5 branches. In the branch which starts at D, there are 4 twigs, two of which, 6 and 7, are from the sub-branch starting at F. For this parton configuration, we will look at two examples of CS states to see how their λ measures differ from λ_0 . The first one is: $(q123\bar{q})(456789)$, which is shown in Fig.3a. The length of the dashed line shows the λ surplus compared to λ_0 . The surplus is denoted by $\delta\lambda$: $\delta\lambda = 2|DH|$. The second CS state is: $(q1234569\bar{q})(78)$ shown in Fig.3b. The λ surplus is: $\delta\lambda = 2(|FG| + |DF| + |HI| + |DH|)$. Of course, the first CS state is more favored than the second one though it has more gluons in its gluon sub-singlet. If we look at them more carefully, we will find that the gluon sub-singlet of the first CS state is made up of all gluons from two adjacent branches, while the sub-singlet of the second is formed by part of gluons in the two branches. We also see that the shorter the line $|DH|$ is, the larger production weights the CS states have. Here $|DH|$ is just the rapidity difference between the two adjacent branches. We see that those CS states with their gluon sub-singlets formed by adjacent twigs (note that twig is the smallest branch) have large production rate. In this case most gluons in gluon sub-singlets are soft ones.

We summarize the above arguments that most of CS states produced in the process have the following configurations:

- The gluon sub-singlet of a CS state is made up of a series of neighboring gluons. If part of a branch joins the sub-singlet, the rest of this branch should also join it. Any separation of one part from the rest of the same branch would result in the suppression of the production weight. The whole branch is like a skeleton. The principle says, if the skeleton is in, the whole body should be in.
- The rapidity difference between two branches at the two ends of the continuous branch sequence is as small as possible. This implies that all branches are as far from the root (or beginning) of the cascade tree as possible. Hence those CS states with their gluon sub-singlets formed by adjacent twigs have large production rate.

Now we begin to discuss our Monte Carlo results which are shown in Fig.4-7. Fig.4 and 5 show how CS states influence the global properties of events, while Fig.6(a-d) and 7(a-d) shows other properties in the case of the T-weight and the constant weight respectively: the distribution of the gluon number in the gluonic sub-singlet of a CS state, the multiplicity distribution in the whole and the restricted phase space, and the rapidity distribution for c-events in the central region. The constant weight is defined as that each possible CS state is chosen with the same probability, and the T-weight is discussed earlier in this section. We show three groups of results: no CS states, 100% and 30% CS states. These results show that for the situation of the constant weight, there is observable though small deviation between the result with CS states and that without CS states, while for the situation of the T-weight, the deviation is even smaller. One may easily accredit the small deviation in event properties to the $1/N_c^2$ effect of the CS state. This is not truly the case. Here two questions should be distinguished: (a) How large is the probability for CS states to appear? and (b) How large is the deviation between the hadronic events with and without CS states included? They are two distinct concepts by nature. We recall that there are many non-orthogonal leading CS states corresponding to an ordered trace state. Though a single leading CS state contribute only $1/N_c^2$, the total contribution may actually be large for many CS states. In fact we cannot determine the total probability at the perturbation level as shown earlier in the former section. Hence we include a parameter R to denote the total probability of CS states which we will give results in two cases R=30. Note that all the above statements are refer to question (a). But when we talk about the small deviation between events with and without CS states, we are addressing question (b). One can understand this deviation as the result of the soft gluon dominance in the cascade and the infrared property of global observables, considering that the global shapes are mainly determined by the cascade instead of the hadronization and that the effect caused by isolating a group of gluons (mostly soft continuous gluons in the color sequence) should not be large. We know earlier in this section and later in the following paragraph that T-weight favors the CS states with softer and less gluons in their gluonic subsinglets which bring little change to global and local observables, while the constant weight gives each CS state equal probability which leads to larger contribution from those CS states with harder and more gluons in their gluonic subsinglets whose observables may differ significantly from those conventional events without CS states. So on average the constant weight gives larger difference from conventional events than the T-weight.

It is meaningful to investigate local observables especially in carefully chosen phase space regions and types of events, where the difference between CS events and Non-CS ones may be more clearly seen. Fig.6a shows the probability distribution of gluon number in the gluon type sub-singlet of a CS state. The T-weight sampling is used to select a CS state for a given parton configuration. The distribution shows a perfect exponential decreasing feature: $P(n_g) \sim \exp(-0.9n_g)$. The mean value of the gluon number $\langle n_g \rangle \sim 2.9$. Fig.7a is the n_g distribution with the constant weight. Its decreasing behavior is much slower than that with the T-weight. It has the mean value $\langle n_g \rangle \sim 3.5$. We know that the T-weight favors the CS state with soft gluons in its gluonic subsinglet. More gluons are in its gluonic subsinglet, more probably it gives a smaller T-weight. Hence the T-weight favors the smaller number

of gluons in the gluonic subsinglet of a CS state. Therefore the $\langle n_g \rangle$ distribution with the T-weight is steeper than that with the constant weight. Fig.6b is the multiplicity distribution for charged particles using the T-weight. The solid line is the result without CS states, and the dashed line is that CS states are produced with the total fraction of 30%. We see that there is little difference after introducing CS states. It is not surprising because we know that in most cases the gluonic sub-singlet of a CS state is composed of soft gluons, which brings little change to the multiplicity of the original string connection. Fig.7b is the multiplicity distribution for charged particles in the constant weight. An apparent deviation from that without CS states can be seen. The broader distribution gives a larger mean multiplicity than in the T-weight case. This phenomenon is consistent with our above analysis that compared to the T-weight case, the constant weight enhances the probability of CS states with harder gluonic subsinglets which give rise to larger multiplicity. Fig.6c is the multiplicity distribution of charged particles in the T-weight in the rapidity range $-1 < y < 0$ for the c-events. The rapidity axis is chosen along the added $c\bar{c}$ momentum. The selection criterion is that the angle of $c\bar{c}$ is smaller than 110° . The solid line is the result without CS states, and the dashed and dotted line are that CS states are produced with total fractions of 100% and 30%, respectively. The reason for choosing the c-event is that for these events it is easy to tag the primary quarks. Normally there should be a rapidity gap caused by CS states between the $c\bar{c}$ cone and the cone in the opposite direction. That is to say, in the rapidity region around $y \sim 0$, there are fewer particles than the case without CS states. This effect can be seen in the figure. The probability to have fewer than 2 charged particles in the range $-1 < y < 0$ in the case with CS states produced is obviously larger than that without them. This is just what we expect a sign (though a weak one) of a depleting area or a rapidity gap. Fig.7c is the result with the constant weight. No sign (even a weak one) of a depleting area is found. This is possibly due to the broader multiplicity distribution brought by the constant weighting (see also Fig.7b). Fig.6d is the rapidity distribution of charged particles for the c-event in the T-weight. The rapidity axis is chosen along the added $c\bar{c}$ momenta. The selection criterion is that the angle of $c\bar{c}$ is smaller than 110° . The solid line is the result without CS states, and the dashed line is that CS states are produced with total fraction of 30%. The difference of the largest and the most interest is in the region $-2 < y < 0$, where the distribution with CS states is a little lower than that without them. This is a weak indication for the particle deficiency in the central region. This phenomenon is absent for Fig.7d where the constant weight is used. On the contrary, the central region of the distribution with CS states is clearly higher than that without them, which implies a more populated central area exists for the case of CS states.

In the literature, there are a number of reconnection models which can describe the production of CS states in a variety of reactions [7,8,12]. Ref. [12] gives a reconnection model (which we call RMDCM for short) which is incorporated into the dipole cascade Monte Carlo ARIADNE. In some sense, its strategy of color reconnection is similar to ours, but it is more complicated and more phenomenologically based. Our model provide a significantly simpler recipe for reconnection. The results for e^+e^- annihilation produced by our model and RMDCM are similar.

Noticing that the CS states with soft-gluon sub-singlet have large T-weights and hence have large production rates, the results of our model have some similarities with those from the Soft Color Interaction (SCI) model [21]. We will in the future study implications from our model for deep inelastic scattering.

VI. SUMMARY

We use the method of color effective Hamiltonian H_c to study properties of color separate states in $e^+e^- \rightarrow q\bar{q} + ng$. The full physical color state is $|H_c\rangle$. There are $n!$ trace terms in $|H_c\rangle$, each corresponds to an order of n gluons. We call each trace term a trace state. Up to $O(1/N_c^2)$, there are $m = \frac{1}{2}n(n-1)$ CSGSs with the leading contribution to a trace state. We obtain the following rules for a leading CSGS: Up to the leading order in $1/N_c$, (a) The state has only two sub-singlets. Each sub-singlet keeps its parton order as that in the trace state at most. (b) Its projection square on the trace state is $1/N_c^2$. (c) The inner product of any two different leading states is $1/N_c^2$, while that of a leading state with a non-leading one is $1/N_c^3$ or higher.

Due to the non-orthogonality of CSGSs, it is necessary to orthogonalize them to give the total probability. Actually the general orthogonalization proves to be extremely difficult. But it is simple and heuristic in some circumstances, e.g. the physical state is fixed at one trace state. In this case, we can estimate the probability of the CSGS through orthogonalization under some further approximations: (a) We take into account leading CSGSs and the singlet chain state with the same gluon order as that of the trace state. (b) We choose a simple form for the transformation matrix. The orthogonalization procedure has two steps. The first step is to find the orthogonal states $\{|s'_i\rangle, i = 1, \dots, m\}$ from m leading CSGSs $\{|s_i\rangle, i = 1, \dots, m\}$. The second step is to introduce the singlet chain state $\{|f\rangle\}$ which is approximately the trace state $\{|h\rangle\}$ into the list, and to obtain the final set of orthogonal states: $\{|f'\rangle, \{|s''_i\rangle, i = 1, \dots, m\}\}$. In the large N_c limit, the probabilities of $|f'\rangle$ and one $|s''_i\rangle$ in the corresponding normalized trace state $|h\rangle$ are given by: $P_{f'} \simeq \frac{1}{1+m/N_c^2}$ and $P_{s''} \simeq (1 - \frac{1}{1+m/N_c^2})/m \simeq 1/N_c^2$. The total probability for all $|s''\rangle$

states are $mP_{s''} \simeq 1 - \frac{1}{1+m/N_c^2} \simeq m/N_c^2$. We see that $P_{f'}$ decreases while $mP_{s''}$ increases monotonously with the growing gluon number (or m), and $P_{f'} + mP_{s''}$ conserves the total probability 1. These results are for large N_c . When N_c is finite, we always span the pole at $m = N_c^2/2$ for $P_{s''}$ which is unphysical. Hence the present result is invalid for finite N_c . In this case, We have to consider all non leading terms in N_c which is too complicated to have an explicit result.

The rules for the leading CSGS are crucial for us to build a practical model to describe the production of color separate states in event generators. Here are the outline of the model:

1. We use a parameter R to describe the total fraction of CS states. We use JETSET to produce a parton configuration with a specific parton connection. We let it correspond to a trace state.
2. We only consider CSGSs with the leading contribution to the trace state. According to the cascade history recorded by JETSET for the current configuration, we can further exclude CS states whose gluon-type sub-singlets are emitted from a single octet gluon.
3. We consider two kinds of weights in selecting individual CS state for an event. One is the constant weight. This is a natural choice if we only consider the color part and neglect the momentum or phase space sector. The other is the T(or λ) measure weight.
4. The fragmentation of the selected CS state is implemented by the Lund string model. Note that the gluon sub-singlet forms a closed string whose fragmentation should be managed with care.

If we use T measure weight, we find that most of CS states produced in the process have the following configurations: (a) The gluon-type sub-singlet of a CS state is made up of all gluons in several continuous branches. If part of a branch joins the sub-singlet, other part of this branch should also join it. Any separation of one part from the rest of the same branch would result in the suppression of the production weight. (b) CS states with their gluon-type sub-singlets formed by adjacent twigs of the branching tree have large production rate. From these points, we see that, CS states whose gluon-type sub-singlets consist of soft adjacent gluons have relatively larger T-weights than other types of CS states and therefore have larger production rates.

The comparisons of global and local event properties between events with and without CS states in the T- and the constant weight are given. Generally the difference of results between two cases, those with and without CS states, is found to be small, though we find the deviation in the constant weight is a little larger than that in the T-weight, and in the central phase space area a weak sign of the rapidity gap is shown in the T-weight case. The main reason lies in the fact that soft gluons dominate in the cascade. Considering the infrared safe properties for all observables, the effect caused by isolating a group of gluons (mostly soft continuous gluons in the color sequence) should not be large. T-weight favors CS states with soft gluonic sub-singlets which should not bring significant change to global and local observables, while the constant weight gives each CS state equal probability which enhances contributions from those CS states with harder gluons in their sub-singlets whose observables may differ significantly from conventional states without separate subsinglets. So on average the constant weight gives rise to larger deviation from conventional events than the T-weight. Conclusively we find that in the high energy electron-positron annihilation, global and local properties are mainly determined by the momentum configuration of final partons, and are almost insensitive to the way of color connection.

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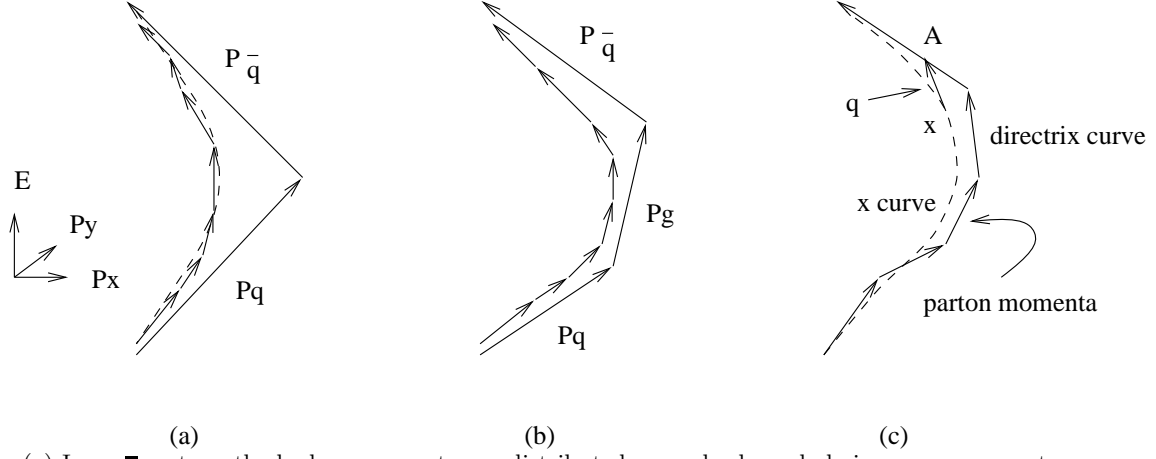


FIG. 1. (a) In a $q\bar{q}$ system, the hadron momenta are distributed around a hyperbola in energy-momentum space. (b) For a $q\bar{q}g$ system, the hadron momenta are distributed around two hyperbola. (c) For a multigluon state, the hadron momenta are distributed around a curve in energy-momentum space (called the x-curve) which smoothly follows the (color-ordered) parton momenta.

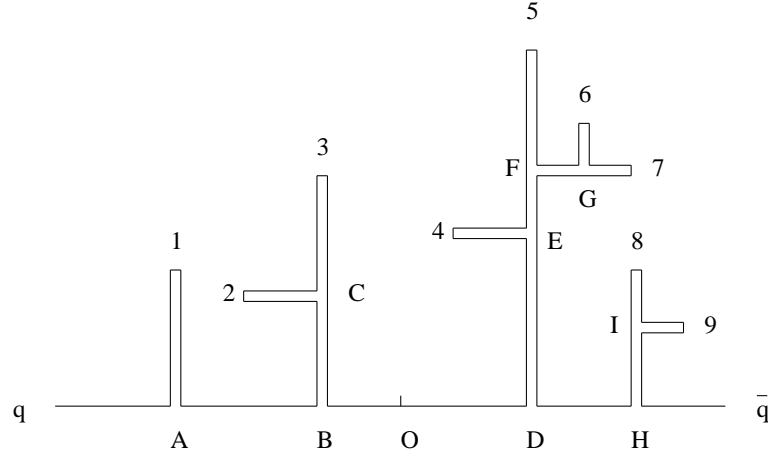


FIG. 2. Diagram of the effective rapidity range (i.e. λ) for a parton configuration.

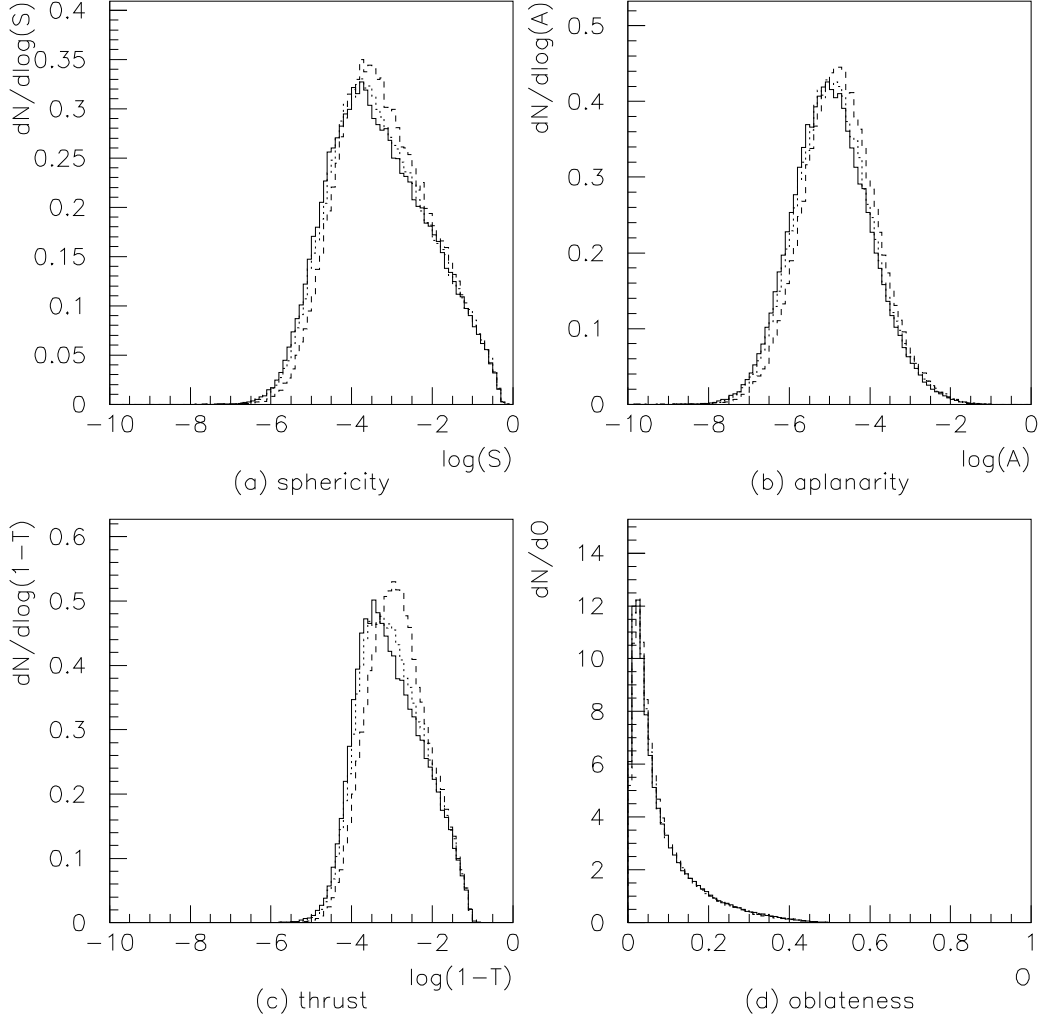


FIG. 4. This figure is for comparing global quantities of events (sphericity, aplanarity, thrust and oblateness) among cases where CS states are produced with 0%, 100% and 30% probability. The CS states are chosen with constant (or equal) weight. The energy is 91 GeV. Solid line: no CS states; Dashed line: 100% CS states. Dotted line: 30% CS states.

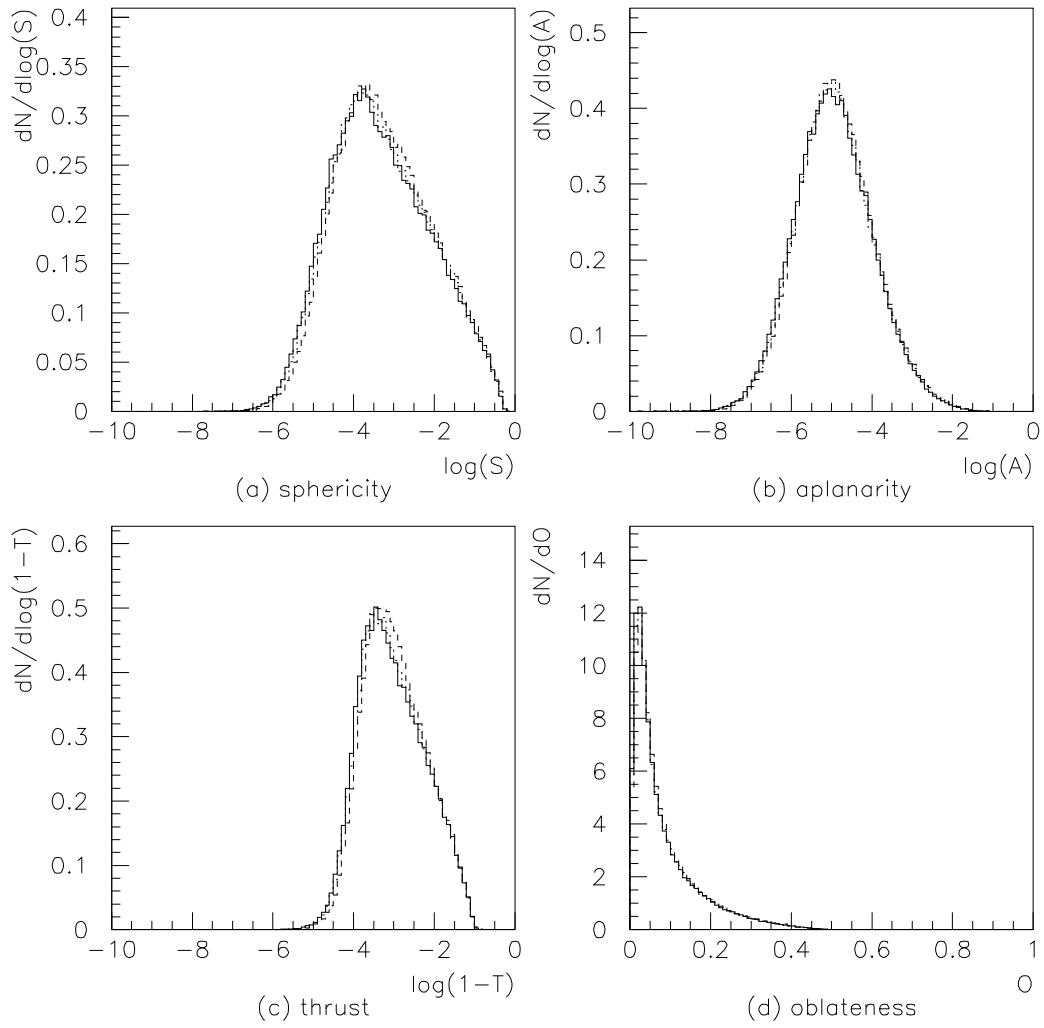


FIG. 5. Same as Fig.4 except that the CS states are chosen with T-weight.

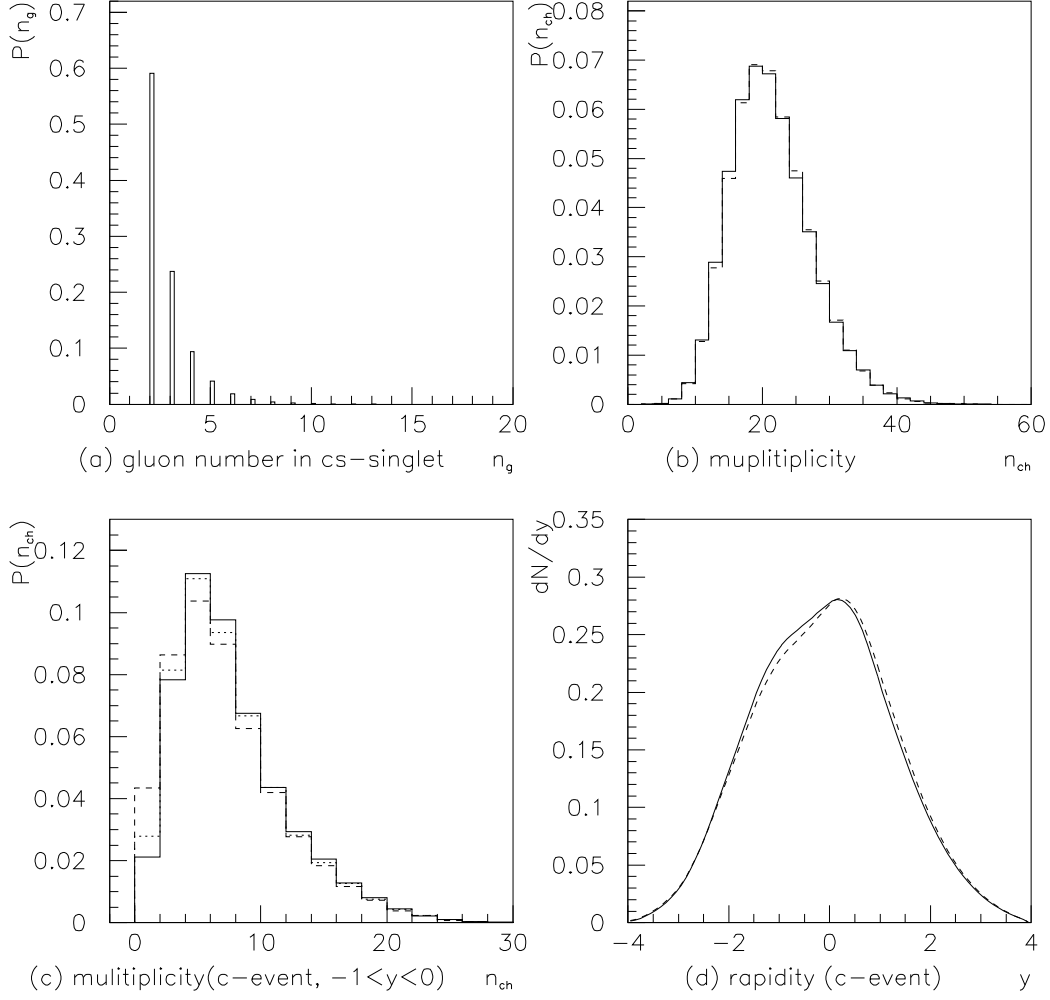


FIG. 6. The energy is 91 GeV. T-weight is used to select CS states. (a) The probability distribution of gluon number in the gluon type sub-singlet of the CS state. (b) Multiplicity distribution of charged particles. Solid line: no CS states; Dashed line: 30% CS states. (c) Multiplicity distribution of charged particles in the rapidity range $-1 < y < 0$ for c -events (i.e. the initial quark-antiquark pair produced at the electroweak vertex is $c\bar{c}$). The rapidity axis is chosen along the added $c\bar{c}$ momentum. The selection criterion is that the angle of $c\bar{c}$ is smaller than 110° . Solid line: no CS states; Dashed and dotted line: 100% and 30% CS states, respectively. (d) Rapidity distribution of charged particles for c -events. The rapidity axis is chosen along the added $c\bar{c}$ momenta. The selection criterion is that the angle of $c\bar{c}$ is smaller than 110° . Solid line: no CS states; Dashed line: 30% CS states.

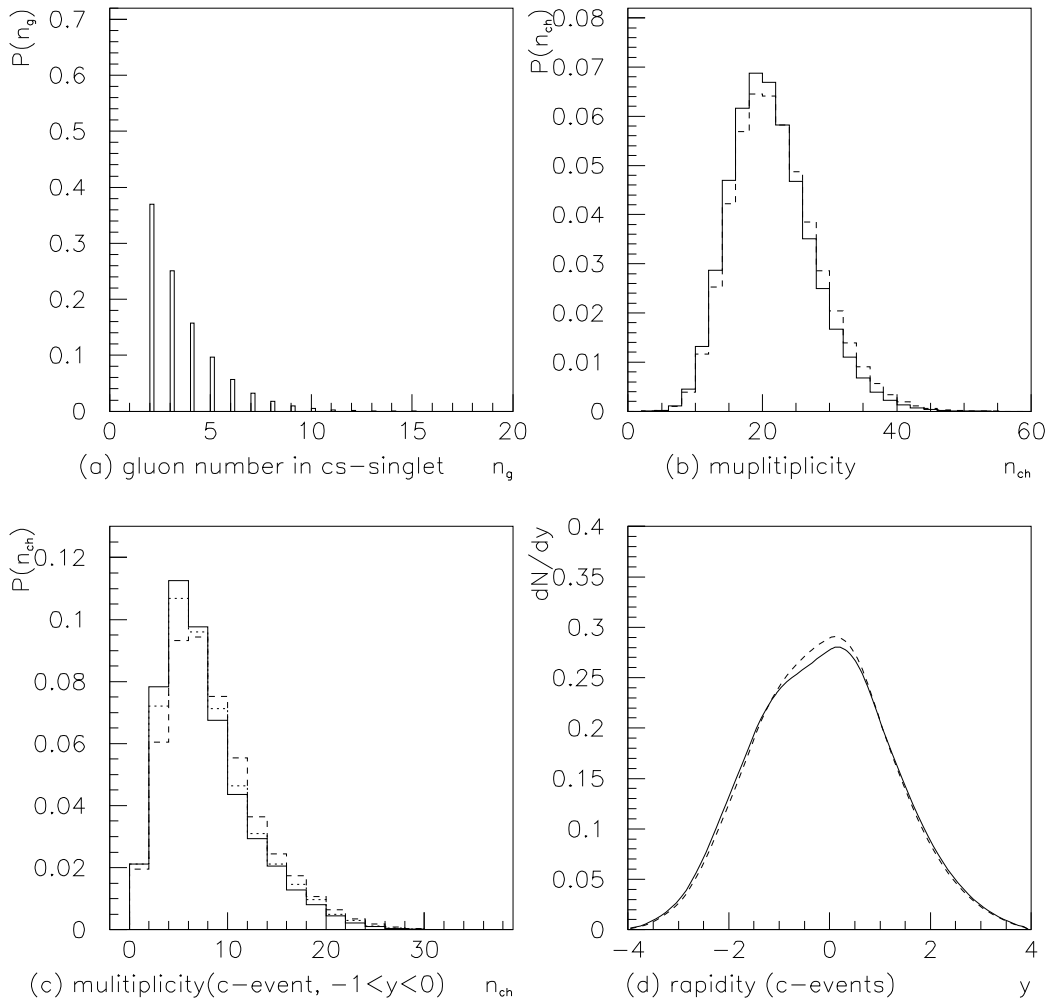


FIG. 7. The same as Fig.6 except that the constant weight is used in selecting CS states.

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